ARLCB-TR-79019

A COMPUTER PROGRAM AND APPROXIMATE SOLUTION FORMULATION FOR GUN MOTIONS ANALYSIS

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June 1979

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AMCMS No. 6111-01-91A0011

PRON No. 1A92ZA01NMLC

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--      1    OF      1
-- 1 -  AD NUMBER: A074935
--48 -  SBI SITE HOLDING SYMBOL:      RSIH
--                                    TTR
-- 2 -  FIELDS AND GROUPS: 12/5, 19/10
-- 3 -  ENTRY CLASSIFICATION:   UNCLASSIFIED
-- 5 -  CORPORATE AUTHOR: ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
--      WATERVLIET NY LARGE CALIBER WEAPON SYSTEMS LAB
-- 6 -  UNCLASSIFIED TITLE:     A COMPUTER PROGRAM AND APPROXIMATE SOLUTION
--      FORMULATION FOR GUN MOTIONS ANALYSIS,
-- 8 -  TITLE CLASSIFICATION:   UNCLASSIFIED
--10 -  PERSONAL AUTHORS: WU,J. J. ;
--11 -  REPORT DATE:      JUN   , 1979
--12 -  PAGINATION:     31P   MEDIA COST:  $  6.00 PRICE CODE:   AA
--14 -  REPORT NUMBER:    ARLCB-TR-79019
--18 -  MONITOR ACRONYM:  SBI
--19 -  MONITOR SERIES:   AD-E440 039
--20 -  REPORT CLASSIFICATION:  UNCLASSIFIED
--23 -  DESCRIPTORS:      *INTERIOR BALLISTICS, *COMPUTER PROGRAMMING, GUN
--      BARRELS, PROJECTILE TRAJECTORIES, FINITE ELEMENT ANALYSIS, MOTION,
--      APPROXIMATION(MATHEMATICS), BOUNDARY VALUE PROBLEMS, CORRECTIONS
--24 -  DESCRIPTOR CLASSIFICATION:    UNCLASSIFIED
--         <<P FOR NEXT PAGE>> OR <<ENTER NEXT COMMAND>>
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1a ROW=24 COL= 67         <Ctrl>H For Help                          N  Poll
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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: ARLCB-TR-79019	2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle): A COMPUTER PROGRAM AND APPROXIMATE SOLUTION FORMULATION FOR GUN MOTIONS ANALYSIS	5. TYPE OF REPORT & PERIOD COVERED
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): J. J. Wu	8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Benet Weapons Laboratory, Watervliet Arsenal, Watervliet, N.Y. 12189, DRDAR-LCB-TL	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: AMCMS No. 6111-01-91A0011, PRON No. 1A92ZA01NMLC
11. CONTROLLING OFFICE NAME AND ADDRESS: US Army Armament Research and Development Command, Large Caliber Weapon Systems Laboratory, Dover, New Jersey 07801	12. REPORT DATE: June 1979
	13. NUMBER OF PAGES: 31
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)	15. SECURITY CLASS. (of this report): UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

To be published in an open literature journal.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Finite Element
Gun Barrel Motions
Gun Dynamics
Internal Ballistics

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The purpose of this paper is to describe some of the features associated with a finite element computer program for approximate solutions of a gun dynamics problem. The lateral motion of a gun tube is modeled by an Euler-Bernoulli beam. The difficulties of the problem are due to various complicated loadings and support conditions which can be nonconservative, highly discontinuous and time dependent. The solution formulation for this generally non-self-adjoint problem has been presented in an earlier paper. In

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terms of finite element discretization, the two-dimensional shape function of spatial and time coordinates is chosen as a product of two one-dimensional shape functions; each for its respective coordinate and both being Hermitian polynomials. The generalized coordinates are then the displacement, slope, velocity and time derivatives of the slope at each node point. The correspondence between local and global generalized coordinates is described. The "stiffness matrices" of spatial and time-effect, contributed by the recoil force, pressure and curvature induced force and the moving mass of a projectile are derived. It is interesting to observe that the strong discontinuities associated with these forces disappear as a result of the smoothing effect of integration in spatial as well as in time coordinates. The present approach to deal with the moving support problem efficiently is also pointed out in this paper. Numerical results of a demonstrative problem are presented.

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A COMPUTER PROGRAM AND APPROXIMATE SOLUTION FORMULATION FOR GUN MOTIONS ANALYSIS

ABSTRACT. The purpose of this paper is to describe some of the features associated with a finite element computer program for approximate solutions of a gun dynamics problem. The lateral motion of a gun tube is modeled by an Euler-Bernoulli beam. The difficulties of the problem are due to various complicated loadings and support conditions which can be nonconservative, highly discontinuous and time dependent. The solution formulation for this generally non-self-adjoint problem has been presented in an earlier paper. In terms of finite element discretization, the two-dimensional shape function of spatial and time coordinates is chosen as a product of two one-dimensional shape functions; each for its respective coordinate and both being Hermitian polynomials. The generalized coordinates are then the displacement, slope, velocity and time derivatives of the slope at each node point. The correspondence between local and global generalized coordinates is described. The "stiffness matrices" of spatial and time-effect, contributed by the recoil force, pressure and curvature induced force and the moving mass of a projectile are derived. It is interesting to observe that the strong discontinuities associated with these forces disappear as a result of the smoothing effect of integration in spatial as well as in time coordinates. The present approach to deal with the moving support problem efficiently is also pointed out in this paper. Numerical results of a demonstrative problem are presented.

1. INTRODUCTION.

It is our ultimate goal to design a gun system from which a projectile can hit a target with precision. The technology involved in this task covers that of the interior and exterior ballistics. As it is obvious from the sequence of events, the position, direction and the velocity of a projectile as it leaves the muzzle, consititue the initial conditions for the problem of exterior ballistics. Clearly then, the capability of providing these data under a variety of loading, supporting and detonating conditions is one of the essential links for the complete chain of designing a precision gun system.

This paper reports some of our results in the initial effort on the gun motions analysis. Specifically, these are some of the detailed features related to a computer program for such an analysis. The finite element method in a very general sense has been chosen here due to its proven flexibility and adaptability for a very wide range of situations in terms of geometry, loading and supporting conditions. Even at this early stage,

we have observed some unusual features of the problem which render the use of "canned" finite element computer program meaningless. First, there is no satisfactory finite element computer program for initial value programs at the present time other than the methodology developed by this writer using an unconstrained variational formulation [1]. The forces considered in the present study include the moving mass effect of the projectile, the curvature and pressure induced load and the recoil force which is nonconservative in nature if the support at the breech end is less than rigid. Various elastic supports are considered. In the due course, effects of frictional forces, large deformation and time-dependent support, and other effects will be included. Therefore, it is only sensible to develop right from the beginning a finite element computer program codes uniquely suitable for the problem on hand, which is capable of handling nonconservative forces, moving mass effects, various supporting conditions including moving supports. In addition, it should also be easily amenable for modifications and inclusions of other effects such as frictional forces, large deformation, and so forth.

The unconstrained variational formulation for approximate solutions of boundary value problems, in conjunction with the finite element descritization has proved to be highly efficient, easy to use and utterly flexible for problems involving nonconservative forces, various support conditions and damping effects [2,3]. Its unique advantage in dealing with initial value problems has also been demonstrated [1]. Therefore it is natural that this finite element unconstrained variational formulation be adapted for the complicated transient problem of gun motions analysis.

The basis of the present formulation for more general cases has been given in a previous paper [4]. The special problem of a uniform gun tube is treated here for demonstrated purposes. The differential, initial and boundary conditions are given in Section 2. This differential equation differs from those used by many other writers by one term. This extra term included in previous papers turned out to be incorrect and the reason is given in Section 3. An unconstrained variational statement which is equivalent to the given governing equation is stated in Section 4. The details of some of the special features on finite element descritization are described in Section 5. Finally some preliminary data which shows the recovery of the initial data are presented in Section 6.

2. GOVERNING EQUATIONS. The motion of a gun tube modeled by the lateral deflection of an Euler-Bernoulli beam is shown in Figure 1. The differential equation in nondimensional form is

$$\begin{aligned} y'''' + (-\bar{P} + g\sin\alpha)[(1-x)y']' + \ddot{y} &= -\bar{P}y''H(\bar{x}-x) \\ &- m[\beta^2t^2y'' + 2\beta t\dot{y}' + \ddot{y}]\bar{\delta}(\bar{x}-x) \\ &- (gm\cos\alpha)\bar{\delta}(\bar{x}-x) - g\cos\alpha \end{aligned} \tag{2-1}$$

with

$$\bar{P} = \pi R^2 p$$

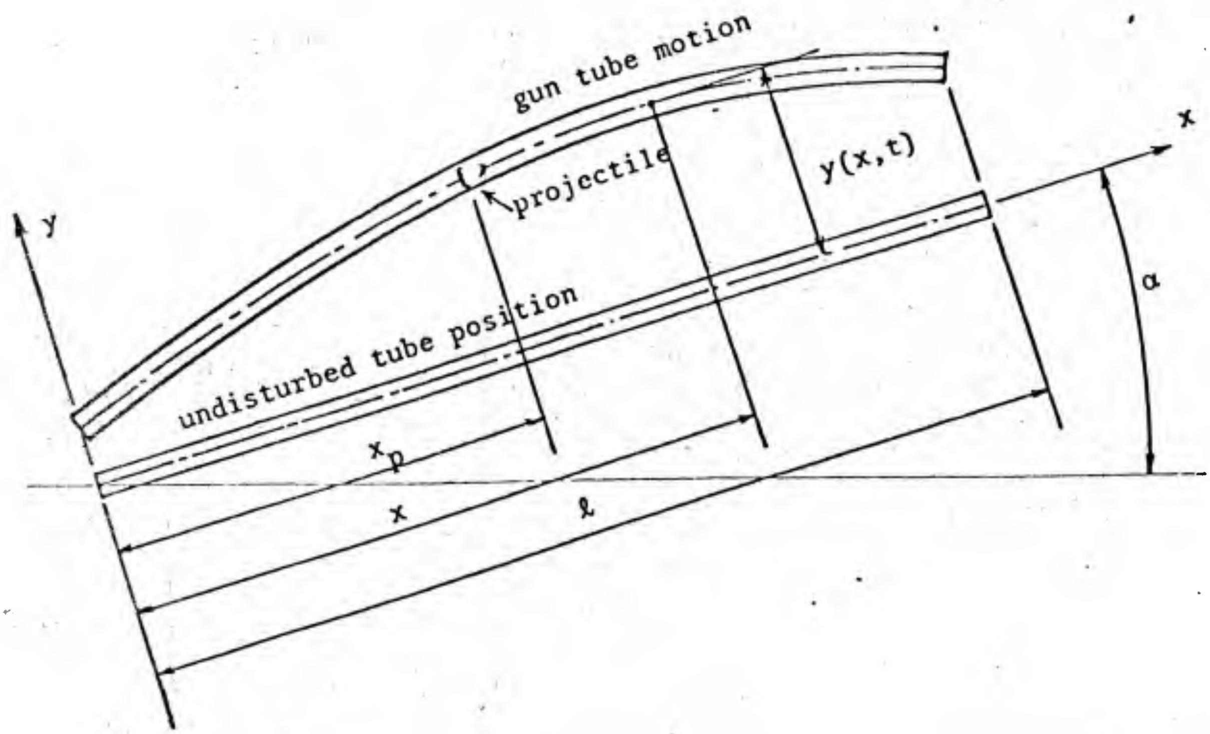


FIGURE 1. A Schematic Drawing of the Problem Configuration.

where

$y = y(x,t)$, the tube deflection
x, spatial axis along the tube's length, $0\underline{<}x\underline{<}1$
t, time axis, $0\underline{<}t\underline{<}T$, T is the time limit of interest
α, elevation angle
m, projectile mass
β, acceleration of the projectile, assumed to be constant
p, bore pressure, assumed to be constant
g, gravitational acceleration

$\bar{x} = \frac{1}{2}\beta t^2$, projectile position

$\bar{H}(x)$, the Heavenside step function
$\bar{\delta}(x)$, the Dirac delta function

In (2-1), a prime (') denotes a differentation with respect to x and dot (˙), a differentiation with respect to t. The derivation of this equation and the end conditions which follow have been given previously [4,5] and will not be repeated here. However, the absence of a term in the present equation will be explained in the next Section.

The initial condition, or more appropriately, the end conditions in time are

$$\left.\begin{aligned} \dot{y}(x,0) &= 0 \\ \dot{y}(x,T)[1 + m\bar{\delta}(\tfrac{1}{2}\beta T^2-x)] + k_7[y(x,0) - Y(x)] &= 0 \end{aligned}\right\} \qquad (2\text{-}2)$$

where the constant k_7 is introduced in conjunction with the unconstrained variational formulation (Section 4) so that if one takes k_7 to be infinite, the initial displacement $y(x,0)$ is forced to be identical to the prescribed shape $Y(x)$. Based on similar reasonings the boundary conditions have been shown to be the following.

$$\left.\begin{aligned} y''(0,t) - k_2 y'(o,t) &= 0 \\ y''(1,t) + k_4 y'(1,t) &= 0 \\ \text{and} \qquad y'''(0,t) + k_1 y(0,t) & \\ + (-\bar{P} + g\cos\alpha)y'(0,t) + \bar{P}y'(0,t)\bar{H}(\tfrac{1}{2}\beta t^2) & \\ + m\beta^2 y'(0,t)\bar{\delta}(\tfrac{1}{2}\beta t^2) &= 0 \\ y'''(1,t) - k_3(1,t) + \bar{P}y'(1,t)\bar{H}(\tfrac{1}{2}\beta t^2-1) & \\ + m\beta^2 y'(1,t)\bar{\delta}(\tfrac{1}{2}\beta t^2-1) &= 0 \end{aligned}\right\} \qquad (2\text{-}3)$$

where k_i, $i = 1,2,3,4$, are the appropriate elastic spring constants at the supports.

3. AN ERROR IN PREVIOUS LITERATURE*. We have already mentioned that there should be a correction made to the force terms due to the projectile's moving mass. This correction will be explained in the present section.

The geometry of a concentrated mass moving on a "beam" structure is illustrated in Figure 2. In this figure, $y(x,t)$ is the beam deflection. The position vector $\underset{\sim}{r}$ of the moving mass m can be written as

$$\underset{\sim}{r} = \xi(t)\underset{\sim}{i} + y(\xi(t),t)\underset{\sim}{j} \tag{3-1}$$

where i,j are the unit vectors in s and y directions respectively; $\xi(t)$ denotes the position of m in x direction. It is easy to derive the velocity and acceleration of $\underset{\sim}{r}$.

$$\left.\begin{aligned} \underset{\sim}{v} &= \frac{d\underset{\sim}{r}}{dt} = \dot{\xi}(t)\underset{\sim}{i} + [\dot{\xi}(t)y'(\xi(t),t) + \frac{\partial y}{dt}]\underset{\sim}{j} \\ \underset{\sim}{a} &= \frac{d\underset{\sim}{v}}{dt} = \ddot{\xi}(t)\underset{\sim}{i} + [\ddot{\xi}(t)y'(\xi,t) + \dot{\xi}^2(t)y''(\xi,t) \\ &\quad +2\dot{\xi}(t)\frac{\partial^2 y}{\partial\xi\partial t} + \frac{\partial^2 y}{\partial t^2}]\underset{\sim}{j} \end{aligned}\right\} \tag{3-2}$$

In deriving (3-2), it should be observed that

$$\begin{aligned} \frac{d}{dt}y(\xi(t),t) &= \frac{\partial y}{\partial\xi}\frac{d\xi}{\partial t} + \frac{\partial y}{\partial t} \\ &= \dot{\xi}\frac{\partial y}{\partial\xi} + \frac{\partial y}{\partial t} = \dot{\xi}\, y' + \frac{\partial y}{\partial t} \end{aligned} \tag{3-3}$$

*This error was first observed by T. E. Simkins of Benet Weapons Laboratory at the Second U.S. Army Symposium on Gun Dynamics, 19-22 September 1978, Rensselaerville, New York.

Since $\underset{\sim}{a}$ is the acceleration of the projectile mass m, the force acting on it must be

$$\left.\begin{aligned} \underset{\sim}{F} &= m\underset{\sim}{a} \\ \text{and} \quad F_x &= ma_x = m\ddot{\xi}(t) \\ F_y &= ma_y \\ &= m\{\ddot{\xi}y' + \dot{\xi}^2 y'' + 2\dot{\xi}\dot{y}' + \frac{\partial^2 y}{\partial t^2}\} \end{aligned}\right\} \qquad (3\text{-}4)$$

It appears that most of the previous investigators have accomplished this much. Now comes the important step, since we are interested in the force imparted on the beam by the moving mass m. This force, from Figure 2, is clearly not due to F_y alone, rather is the sum of the components of F_x and F_y in the normal direction of the beam. Denote this sum by F_m, one has

$$F_m = F_y \cos\theta - F_x \sin\theta \qquad (3\text{-}5)$$

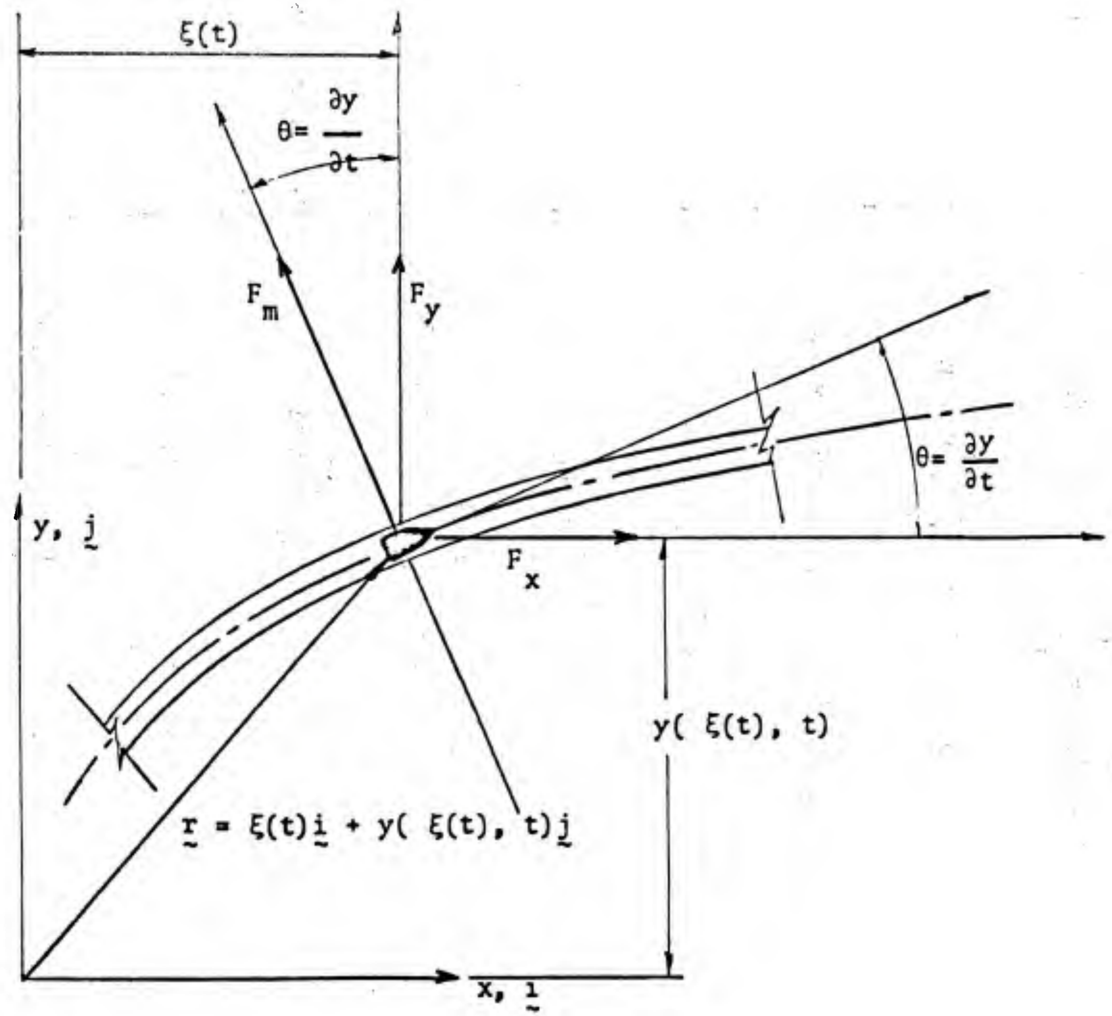
But, for small deflection, θ is small so that

$$\left.\begin{aligned} \cos\theta &\cong 1 \\ \sin\theta &\cong \tan\theta \cong \theta = \frac{\partial y}{\partial \xi} = y' \end{aligned}\right\} \qquad (3\text{-}6)$$

Substitute (3-6) into (3-5), one obtains

$$\left.\begin{aligned} F_m &= F_y - m\ddot{\xi}y' \\ \text{or} \quad F_m &= m\{\dot{\xi}^2 y'' + 2\dot{\xi}\frac{\partial y'}{\partial t} + \frac{\partial^2 y}{\partial t^2}\} \end{aligned}\right\} \qquad (3\text{-}7)$$

Eq. (3-7) is the correct expression for the force imparted on a carrying structure due to a moving mass. This structure can be a rail, a bridge or, as in the present case, a gun tube. It is somewhat surprising to find that the expression for F_y of Eq. (3-4) rather than F_m of Eq. (3-7) is used in the literature known to this writer (see, for example, [6]).



<u>FIGURE 2</u>. Forces Acting Between Gun Tube and a Moving Projectile.

4. UNCONSTRAINED VARIATIONAL STATEMENT. Through integrations-by-parts, it is an easy matter to show that the following variational statement is equivalent to the differential equation and ends conditions stated in Section 2.

$$\delta I = (\delta I)_y = \sum_{i=1}^{13} (\delta I_i)_y - \sum_{j=1}^{3} (\delta J_j) = 0 \tag{4-1}$$

with

$$\left.\begin{aligned} J_1 &= -\, Tg \cos\alpha \int_0^1 \int_0^1 y^* dx dt \\ J_2 &= -\, Tgm \cos\alpha \int_0^1 \int_0^1 y^* \delta(\bar{x}-x) dx dt \\ J_3 &= k_7 \int_0^1 Y(x) y^*(x,1) dx \end{aligned}\right\} \tag{4-2}$$

and

$$\left.\begin{aligned} I_1 &= \int_0^1 \int_0^1 y'' y^{*\prime\prime} dx dt \\ I_2 &= (\bar{P} - g \sin\alpha) \int_0^1 \int_0^1 (1-x) y' y^{*\prime} dx dt \\ I_3 &= -\int_0^1 \int_0^1 \ddot{y} y^* dx dt \\ I_4 &= -\,\bar{P} \int_0^1 \int_0^1 y' y^{*\prime}\, H(\bar{x}-x) dx dt \\ I_5 &= -\,\bar{P} \int_0^1 \int_0^1 y' y^{*\prime} \bar{\delta}(\bar{x}-x) dx dt \end{aligned}\right\} \text{(4-3, first part)}$$

$$I_6 = -\frac{m\beta^2}{T}\int_0^1\int_0^1 t^2 y' y^{*\prime} \bar{\delta}(\bar{x}-x)\,dx\,dt$$

$$I_7 = -\frac{m\beta^2}{T}\int_0^1\int_0^1 t\, y' y^{*\prime} \bar{\delta}'(\bar{x}-x)\,dx\,dt$$

$$I_8 = \frac{2m\beta}{T}\int_0^1\int_0^1 t\dot{y}' y^{*} \bar{\delta}(\bar{x}-x)\,dx\,dt$$

$$I_9 = \frac{m\beta}{T}\int_0^1\int_0^1 y' y^{*} \bar{\delta}(\bar{x}-x)\,dx\,dt$$

$$I_{10} = -\frac{m}{T}\int_0^1\int_0^1 \ddot{y} y^{*} \bar{\delta}(\bar{x}-x)\,dx\,dt \tag{4-3, second part}$$

$$I_{11} = -\frac{m}{T}\int_0^1\int_0^1 \dot{y} y^{*} \dot{\bar{\delta}}(\bar{x}-x)\,dx\,dt$$

$$\begin{aligned} I_{12} = T\int_0^1 \{ & k_1 y(0,t)y^*(0,t) + k_2 y'(0,t)y^{*\prime}(0,t) \\ & + k_3 y(1,t)y^*(1,t) + k_4 y'(1,t)y^{*\prime}(1,t) \\ & + k_5 y(x_s,t)y^*(x_s,t) + k_6 y'(x_s,t)y^{*\prime}(x_s,t)\}dt \end{aligned}$$

$$I_{13} = k_7\int_0^1 y(x,0)y^*(x,1)\,dx$$

This variational statement will serve as the basis of our finite element solutions.

5. FINITE ELEMENT DISCRETIZATION.

5.1 From Local to Global Coordinates. The purpose of the discretization is to enable one to write the variational statement of Eq. (4-1), which is a functional of continuous functions y and y*, etc., in the form of a matrix equation

$$\delta \underset{\sim}{Y}^{*T} \underset{\sim}{K}\, \underset{\sim}{Y} = \delta \underset{\sim}{Y}^{*T} \underset{\sim}{F} \tag{5-1}$$

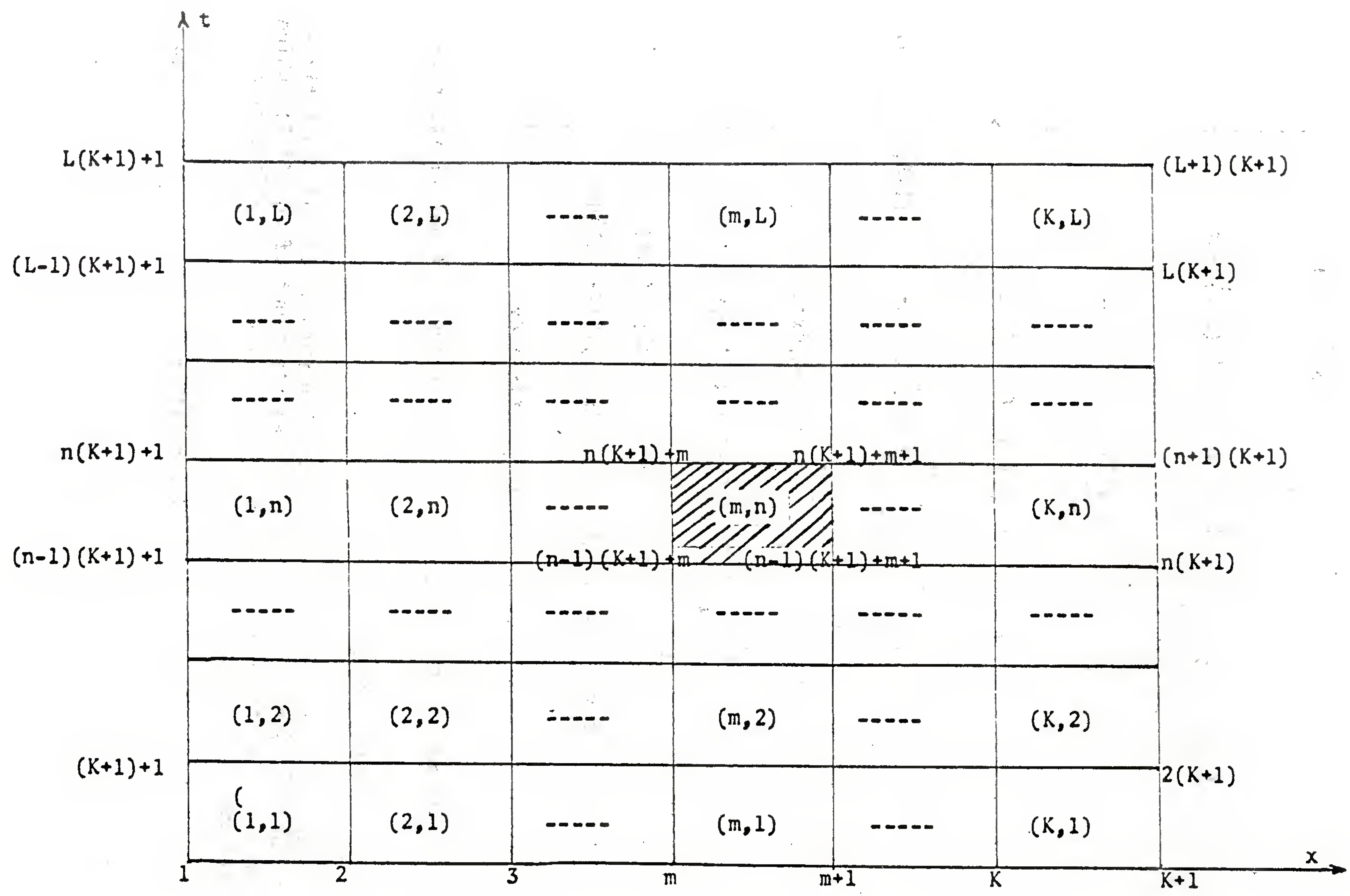


FIGURE 3. Globe Node Scheme for the (m,n)th Element.

where Y, Y* are the "global" generalized coordinates vectors. K is the global "stiffness" matrix, and F the "force" vector. These terminology are patented after the static structural analysis, but they do not necessarily have the physical meanings of those adjectives attached to them. Since the variational statement associated with (5-1) is unconstrained, the equation leads directly to

$$\underset{\sim}{K}\,\underset{\sim}{Y} = \underset{\sim}{F} \tag{5-2}$$

which can be solved for Y if K and F are properly defined. The process by which K and F are assembled and the relation between Y and the desired solution y(x,t) will be described in detail here in this section.

The first step is to write down the expressions in the variational statement in terms of the element variables. A grid scheme of elements is shown in Figure 3. In this figure, the nondimensional length of the gun tube is divided into K equal segments and the time range of interest into L equal segments. The result is then a set of K x L rectangular elements. In the equations that follow n, the sub- or super-scripts m,n denote the association with the m^{th}, n^{th} segments or the $(m,n)^{th}$ element. Define the relation between the local coordinates (ξ,η) of the $(m,n)^{th}$ element and the global coordinates (x,t) by

$$\begin{aligned} \xi &= \xi^{(m)} = Kx - m + 1 \\ \eta &= \eta^{(n)} = Lt - n + 1 \end{aligned} \tag{5-3}$$

Or

$$\begin{aligned} x &= \frac{1}{K}(\xi+m-1) \\ t &= \frac{1}{L}(\eta+n-1) \end{aligned} \tag{5-4}$$

One can write, from Eqs. (4-2) and (4-3):

$$\left.\begin{aligned} \delta I_1 &= \sum_{m=1}^{K}\sum_{n=1}^{L}\frac{TK^3}{L}\int_0^1\int_0^1 y''_{(m,n)}\,\delta y^{*''}_{(m,n)}\,d\xi d\eta \\ \delta I_2 &= \sum\sum\frac{T}{L}(\bar{P}-g\sin\alpha)\int_0^1\int_0^1[(K-m+1)-\xi]\,y'_{(m,n)}\,\delta y^{*'}_{(m,n)}\,d\xi d\eta \end{aligned}\right\} \tag{5-5a}$$

$$\delta I_3 = -\sum\sum \frac{L}{TK}\int_0^1\int_0^1 \dot{y}_{(m,n)}\delta\dot{y}^*_{(m,n)}d\xi d\eta$$

$$\delta I_4 = -\sum\sum \frac{\bar{P}TK}{L}\int_0^1\int_0^1 y'_{(m,n)}\delta y^{*\prime}_{(m,n)}H_{(m,n)}(\bar{\xi}-\xi)d\xi d\eta$$

$$\delta I_5 = \sum\sum \frac{\bar{P}TK}{L}\int_0^1\int_0^1 y'_{(m,n)}\delta y^*_{(m,n)}\bar{\delta}_{(m,n)}(\bar{\xi}-\xi)d\xi d\eta$$

$$\delta I_6 = -\sum\sum \frac{m\beta^2T^3K^2}{L^3}\int_0^1\int_0^1 [(m-1)^2+2(m-1)\eta+\eta^2]y'_{(m,n)}\delta y^{*\prime}_{(m,n)}\bar{\delta}_{(m,n)}(\bar{\xi}-\xi)d\xi d\eta$$

$$\delta I_7 = -\sum\sum \frac{m\beta^2T^3K^2}{L^3}\int_0^1\int_0^1 [(m-1)^2+2(m-1)\eta+\eta^2]y'_{(m,n)}\delta y^{*\prime}_{(m,n)}\bar{\delta}'_{(m,n)}(\bar{\xi}-\xi)d\xi d\eta$$

$$\delta I_8 = \sum\sum \frac{2m\beta TK}{L}\int_0^1\int_0^1 [(m-1)+\eta]\dot{y}'_{(m,n)}\delta y^*_{(m,n)}\bar{\delta}_{(m,n)}(\bar{\xi}-\xi)d\xi d\eta$$

$$\delta I_9 = \sum\sum \frac{m\beta TK}{L}\int_0^1\int_0^1 y'_{(m,n)}\delta y^*_{(m,n)}\bar{\delta}_{(m,n)}(\bar{\xi}-\xi)d\xi d\eta$$

$$\delta I_{10} = -\sum\sum \frac{mL}{T}\int_0^1\int_0^1 \dot{y}_{(m,n)}\delta\dot{y}^*_{(m,n)}\bar{\delta}_{(m,n)}(\bar{\xi}-\xi)d\xi d\eta$$

$$\delta I_{11} = -\sum\sum \frac{mL}{T}\int_0^1\int_0^1 \dot{y}_{(m,n)}\delta y^*_{(m,n)}\dot{\bar{\delta}}_{(m,n)}(\bar{\xi}-\xi)d\xi d\eta$$

$$\delta I_{12} = \sum_{n=1}^{L}\frac{T}{L}\int_0^1 [k_1 y_{(1,n)}\delta y^*_{(1,n)} + k_2K^2y'_{(1,n)}\delta y^{*\prime}_{(1,n)}$$

$$+ k_3 y_{(K,n)}\delta y^*_{(K,n)} + k_4K^2y'_{(K,n)}\delta y^{*\prime}_{(K,n)}]d\eta$$

$$\delta I_{13} = \sum_{m=1}^{K}\frac{k_7}{K}\int_0^1 y_{(m,1)}\delta y^*_{(m,L)}d\xi \qquad (5\text{-}5b)$$

and

$$
\left.
\begin{aligned}
\delta J_1 &= -\sum_{m=1}^{K}\sum_{n=1}^{L}\frac{Tg\cos\alpha}{KL}\int_0^1\int_0^1 \delta y^*_{(m,n)}\,d\xi\, d\eta \\
\delta J_2 &= -\sum\sum\frac{Tg\cos\alpha}{L}\int_0^1\int_0^1 \delta y^*_{(m,n)}\,\bar{\delta}_{(m,n)}(\bar{\xi}-\xi)\,d\xi\, d\eta \\
\delta J_3 &= \sum_{m=1}^{K}\frac{k_7}{K}\int_0^1 Y_{(m)}(\xi)\,\delta y^*_{(m,L)}\,d\xi
\end{aligned}
\right\}
\tag{5-6}
$$

Now, the shape functions are introduced. Let

$$
\begin{aligned}
y_{(m,n)}(\xi,\eta) &= \underset{\sim}{a}^T(\xi,\eta)\underset{\sim}{Y}^{(m,n)} \\
y_{(m,n)}(\xi,\eta) &= \underset{\sim}{a}^T(\xi,\eta)\underset{\sim}{Y}^{*(m,n)}
\end{aligned}
\tag{5-7}
$$

where $a(\xi,\eta)$ is the shape function vector to be chosen and $Y^{(m,n)}$ is the generalized coordinates vector associated with the $(m,n)^{th}$ element. The choice of $a(\xi,\eta)$ and the meaning of $\underset{\sim}{Y}^{(m,n)}$ will now be given. We shall further write

$$
\begin{aligned}
y_{(m,n)}(\xi,\eta) &= \underset{\sim}{a}^T(\xi,\eta)\underset{\sim}{Y}^{(m,n)} = \sum_{k=1}^{16} a_k(\xi,\eta)Y_k^{(m,n)} \\
y^*_{(m,n)}(\xi,\eta) &= \underset{\sim}{a}^T(\xi,\eta)\underset{\sim}{Y}^{(m,n)} = \sum_{k=1}^{16} a_k(\xi,\eta)Y_k^{*(m,n)}
\end{aligned}
\tag{5-8}
$$

Here in this paper, we select $a_k(\xi,\eta)$ as

$$
a_k(\xi,\eta) = b_i(\xi)b_j(\eta), \qquad \begin{matrix} i,j=1,2,3,4 \\ k=1,2,\ldots 16 \end{matrix}
\tag{5-9}
$$

where $b_i(\xi)$ or $b_i(\eta)$ are defined by

$$
\begin{aligned}
b_1(\xi) &= 1 - 3\xi^2 + 2\xi^3 \\
b_2(\xi) &= \xi - 2\xi^2 + \xi^3 \\
b_3(\xi) &= 3\xi^2 - 2\xi^3 \\
b_4(\xi) &= -\xi^2 + \xi^3
\end{aligned}
\tag{5-10}
$$

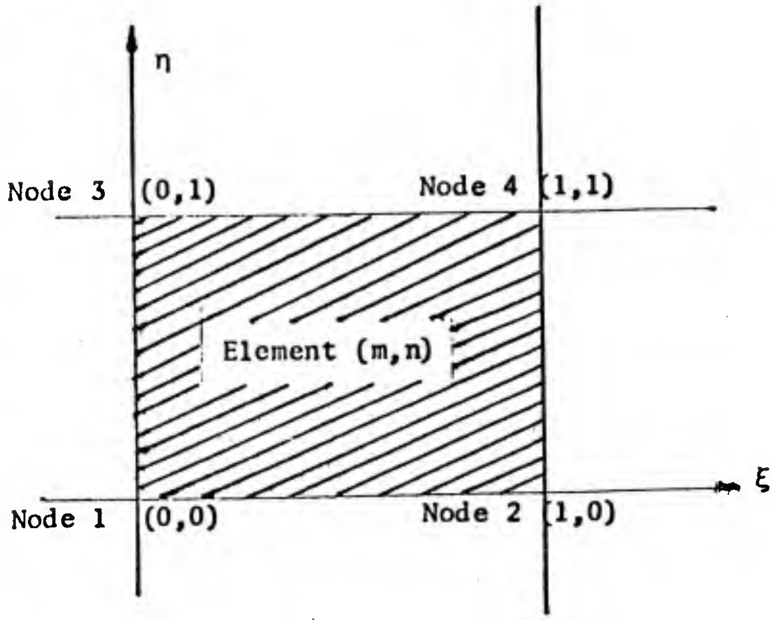


FIGURE 4. Element Node Scheme.

TABLE I. CORRESPONDENCE BETWEEN LOCAL AND GLOBAL NODE NUMBERS

Node Number	
Local	Global
1	(n-1)(K+1)+m
2	(n-1)(K+1)+m+1
3	n(K+1)+m
4	n(K+1)+m+1

Now, the local node scheme is defined in Figure 4 and the correspondence between a local node of the $(m,n)^{th}$ element and a global node is given in Table I.

With Eqs. (5-9) and (5-10) in conjunction of the equation of discretization (5-8), the correspondence between the index k and the pair (i,j) of equation (5-9) and that between $Y_k(m,n)$ and $y_{(m,n)}(\xi,\eta)$ can be easily established. These relations are given in Table II.

In terms of $\underset{\sim}{a}(\xi,\eta)$, Y and Y*, the expressions of δI_i, δJ_i, etc. can be written as

$$\delta I_1 = \sum_{m=1}^{K}\sum_{n=1}^{L} \frac{TK^3}{L}\,\delta Y^{*T}_{(m,n)} \int_0^1\int_0^1 \underset{\sim}{a}_{,\xi\xi}\underset{\sim}{a}^T_{,\xi\xi}\, d\xi d\eta\, \underset{\sim}{Y}_{(m,n)}$$

$$\delta I_2 = \sum_{m=1}^{K}\sum_{n=1}^{L} (\bar{P}-g\sin\alpha)\,\frac{T}{L}\,\delta \underset{\sim}{Y}^{*T}_{(m,n)} \int_0^1\int_0^1 [K-m+1-\xi]\underset{\sim}{a}_{,\xi}\underset{\sim}{a}^T_{,\xi}\, d\xi d\eta\, \underset{\sim}{Y}_{(m,n)}$$

$$\delta I_3 = \sum_{m=1}^{K}\sum_{n=1}^{L} \left(-\frac{L}{TK}\right)\delta \underset{\sim}{Y}^{*T}_{(m,n)} \int_0^1\int_0^1 \underset{\sim}{a}_{,\eta}\underset{\sim}{a}^T_{,\eta}\, d\xi d\eta\, \underset{\sim}{Y}_{(m,n)}$$

$$\delta I_4 = \sum\sum \left(-\frac{TK\bar{P}}{L}\right)\delta \underset{\sim}{Y}^{*T}_{(m,n)} \int_0^1\int_0^1 \underset{\sim}{a}_{,\xi}\underset{\sim}{a}^T_{,\xi}\, H_{(m,n)}(\bar{\xi}-\xi)\, d\xi d\eta\, \underset{\sim}{Y}_{(m,n)}$$

$$\delta I_5 = \sum\sum \left(\frac{TK\bar{P}}{L}\right)\delta \underset{\sim}{Y}^{*T}_{(m,n)} \int_0^1\int_0^1 \underset{\sim}{a}\,\underset{\sim}{a}^T_{,\xi}\, \bar{\delta}_{(m,n)}(\bar{\xi}-\xi)\, d\xi d\eta\, \underset{\sim}{Y}_{(m,n)}$$

$$\delta I_6 = \sum\sum \left(-\frac{m\beta^2T^3K^2}{L^3}\right)\delta \underset{\sim}{Y}^{*T}_{(m,n)} \int_0^1\int_0^1 [(m-1)^2+2(m-1)\eta+\eta^2]\,\cdot$$
$$\cdot\, \underset{\sim}{a}_{,\xi}\underset{\sim}{a}^T_{,\xi}\, \bar{\delta}_{(m,n)}(\bar{\xi}-\xi)\, d\xi d\eta\, \underset{\sim}{Y}_{(m,n)}$$

$$\delta I_7 = \sum\sum \left(-\frac{m\beta^2T^3K^2}{L^3}\right)\delta \underset{\sim}{Y}^{*T}_{(m,n)} \int_0^1\int_0^1 [(m-1)^2+2(m-1)\eta+\eta^2]\,\cdot$$
$$\cdot\, \underset{\sim}{a}\,\underset{\sim}{a}^T_{,\xi}\, \bar{\delta}'_{(m,n)}(\bar{\xi}-\xi)\, d\xi d\eta\, \underset{\sim}{Y}_{(m,n)}$$

$$\delta I_8 = \sum\sum \left(2m\beta\,\frac{TK}{L}\right)\delta \underset{\sim}{Y}^{*T}_{(m,n)} \int_0^1\int_0^1 [(m-1)+\eta]\underset{\sim}{a}\,\underset{\sim}{a}_{,\xi\eta}\, \bar{\delta}_{(m,n)}(\bar{\xi}-\xi)\, d\xi d\eta\, \underset{\sim}{Y}_{(m,n)}$$

(5-11,part A)

$$\delta I_9 = \sum\sum \frac{m\beta TK}{L} \delta Y^{*T}_{\sim(m,n)} \int_0^1 \int_0^1 \underset{\sim}{a}\, \underset{\sim}{a}^T_{,\xi}\, \bar{\delta}_{(m,n)} (\bar{\xi}-\xi) d\xi d\eta\, \underset{\sim}{Y}_{(m,n)}$$

$$\delta I_{10} = \sum\sum \left(-\frac{mL}{T}\right) \delta Y^{*T}_{\sim(m,n)} \int_0^1 \int_0^1 \underset{\sim}{a}_{,\eta} \underset{\sim}{a}^T_{,\eta}\, \bar{\delta}_{(m,n)} (\bar{\xi}-\xi) d\xi d\eta\, \underset{\sim}{Y}_{(m,n)}$$

$$\delta I_{11} = \sum\sum \left(-\frac{mL}{T}\right) \delta Y^{*T}_{\sim(m,n)} \int_0^1 \int_0^1 \underset{\sim}{a}\, \underset{\sim}{a}^T_{,\eta}\, \dot{\bar{\delta}}_{(m,n)} (\bar{\xi}-\xi) d\xi d\eta\, \underset{\sim}{Y}_{(m,n)}$$

$$\delta I_{12} = \sum_{n=1}^{L} \left(\frac{T}{L}\right) \{ \delta Y^{*T}_{\sim(1,n)} [k_1 \int_0^1 \underset{\sim}{a}(0,\eta) \underset{\sim}{a}^T(0,\eta) d\eta$$

$$+ k_2 K^2 \int_0^1 \underset{\sim}{a}_{,\xi}(0,\eta) \underset{\sim}{a}^T_{,\xi}(0,\eta) d\eta] \underset{\sim}{Y}_{(1,n)}$$

$$+ \delta Y^{*T}_{\sim(K,n)} [k_3 \int_0^1 \underset{\sim}{a}(1,\eta) \underset{\sim}{a}^T(1,\eta) d\eta$$

$$+ k_4 K^2 \int_0^1 \underset{\sim}{a}_{,\xi}(1,\eta) \underset{\sim}{a}^T_{,\xi}(1,\eta) d\eta] \underset{\sim}{Y}_{(K,n)} \}$$

(5-11,part B)

$$\delta I_{13} = \sum_{m=1}^{K} \frac{k_7}{K} \delta Y^{*}_{\sim(m,L)} \int_0^1 a(\xi,1) a^T(\xi,0) d\xi\, \underset{\sim}{Y}_{(m,1)}$$

Also

$$\delta J_1 = \sum_{m=1}^{K} \sum_{n=1}^{L} \left(-\frac{Tg\cos\alpha}{KL}\right) \delta Y^{*T}_{\sim(m,n)} \int_0^1 \int_0^1 a(\xi,\eta) d\xi d\eta$$

$$\delta J_2 = \sum_{m=1}^{K} \sum_{n=1}^{L} \left(-\frac{Tgm\cos\alpha}{L}\right) \delta Y^{*T}_{\sim(m,n)} \int_0^1 \int_0^1 \underset{\sim}{a}(\xi,\eta) \bar{\delta}_{(m,n)} (\bar{\xi}-\xi) d\xi d\eta$$

$$\delta J_3 = \sum_{m=1}^{K} \left(\frac{k}{K}\right) \delta Y^{*T}_{(m,L)} \int_0^1 \int_0^1 Y_{(m)}(\xi) \underset{\sim}{a}(\xi,1) d\xi \qquad (5\text{-}12)$$

With δI_i, δJ_i of the variational statement Eq. (4-1) written in terms of $a(\xi,\eta)$, Y and δY^* as given in Eqs. (5-11) and (5-10), we can now discuss the construction of various element matrices and force vectors.

5.2 Common Element Matrices. A common element matrix is one which is the same for all elements. There are four such matrices in this solution formulation and they can be identified in the expressions for δI_1, δI_2, δI_3 in Eqs. (5-11). Let:

$$\left.\begin{aligned}
\delta I_1 &= \sum_{m=1}^{K}\sum_{n=1}^{L} \frac{TK^3}{L}\, \delta \underset{\sim}{Y}^{*T}_{(m,n)}\, \underset{\sim}{B}\, \underset{\sim}{Y}_{(m,n)} \\
\delta I_2 &= \sum_{m=1}^{K}\sum_{n=1}^{L} (P - g\sin\alpha)\, \frac{T}{L}\, \delta \underset{\sim}{Y}^{*T}_{(m,n)} [(K-m+1)\underset{\sim}{A} - \underset{\sim}{D}]\underset{\sim}{Y}_{(m,n)} \\
\delta I_3 &= \sum_{m=1}^{K}\sum_{n=1}^{L} \left(-\frac{L}{TK}\right) \delta \underset{\sim}{Y}^{*T}_{(m,n)}\, \underset{\sim}{C}\, \underset{\sim}{Y}_{(m,n)}
\end{aligned}\right\} \quad (5\text{-}13)$$

Then

$$\left.\begin{aligned}
\underset{\sim}{A} &= \int_0^1\int_0^1 \underset{\sim}{a}_{,\xi}\underset{\sim}{a}^T_{,\xi}\, d\xi d\eta \\
\underset{\sim}{B} &= \int_0^1\int_0^1 \underset{\sim}{a}_{,\xi\xi}\underset{\sim}{a}^T_{,\xi\xi}\, d\xi d\eta \\
\underset{\sim}{C} &= \int_0^1\int_0^1 \underset{\sim}{a}_{,\eta}\underset{\sim}{a}^T_{,\eta}\, d\xi d\eta \\
\underset{\sim}{D} &= \int_0^1\int_0^1 \xi\, \underset{\sim}{a}_{,\xi}\underset{\sim}{a}^T_{,\xi}\, d\xi d\eta
\end{aligned}\right\} \quad (5\text{-}14)$$

Each of these matrices $\underset{\sim}{A}$, $\underset{\sim}{B}$, $\underset{\sim}{C}$ or $\underset{\sim}{D}$ is of size 16 x 16 and thus has 256 values to be evaluated. Since they are independent of the element parameters, they need only to be computed once for all the elements.

5.3 Element Matrices Due to Discontinuous Load Function. These element matrices are identified with the terms of δI_4 through δI_{11} in Eqs. (5-11) where a discontinuous function $H_{(m,n)}(\bar{\xi}-\xi)$, $\delta_{(m,n)}(\bar{\xi}-\xi)$, or their derivative is involved in the integration. It is clear that the line of discontinuity is defined by $\xi = \bar{\xi}(\eta)$. The specific form of $\bar{\xi}(\eta)$ is now given. Since we assume that the projectile moves with a constant acceleration β and that the velocity and position are zero at $t = 0$, the projectile position can be written as

$$x = \bar{x} = \frac{1}{2}\beta t^2 \tag{5-15}$$

From Eqs. (5-4), one has

$$\xi = \bar{\xi} = -m+1+ \frac{\beta T^2 K}{2L^2}(\eta+n-1)^2 \tag{5-16}$$

Eq. (5-16) is obviously dependent on the element parameter m,n. In addition, the integrations of δI_4 through δI_{11}, will depend on whether and how the curve of Eq. (5-15) actually goes through the particular element (m,n). Thus these element matrices must be evaluated element by element.

5.4 Boundary Element Matrices and Non-Self-Adjoint Matrices. The contribution due to the end elements are identified with the δI_{12} and δI_{13} terms. Let

$$\delta I_{12} = \sum_{n=1}^{L} \left(\frac{T}{L}\right) [\delta \underset{\sim}{Y}^{*T}_{(1,n)}[k_1\underset{\sim}{B}_1 + k_2K^2\underset{\sim}{B}_2]\underset{\sim}{Y}_{(1,n)}$$

$$+ \delta\underset{\sim}{Y}^{*T}_{(K,n)}[k_3\underset{\sim}{B}_3 + k_4K^2\underset{\sim}{B}_4]\underset{\sim}{Y}_{(K,n)}]$$

$$\delta I_{13} = \sum_{m=1}^{K} \delta\underset{\sim}{Y}^{*T}_{(m,L)} \frac{k_7}{L} \underset{\sim}{B}_7 \underset{\sim}{Y}_{(m,1)}$$

Then, it is clear from Eqs. (5-12) that

$$\underset{\sim}{B}_1 = \int_0^1 \underset{\sim}{a}(0,\eta)\underset{\sim}{a}^T(0,\eta)d\eta$$

$$\underset{\sim}{B}_2 = \int_0^1 \underset{\sim}{a}_{,\xi}(1,\eta)\underset{\sim}{a}^T_{,\xi}(1,\eta)d\eta$$

$$\underset{\sim}{B}_3 = \int_0^1 \underset{\sim}{a}(1,\eta)\underset{\sim}{a}^T(1,\eta)d\eta$$

$$\underset{\sim}{B}_4 = \int_0^1 \underset{\sim}{a}_{,\xi}(1,\eta)\underset{\sim}{a}^T_{,\xi}(1,\eta)d\eta$$

and

$$\underset{\sim}{B}_7 = \int_0^1 \underset{\sim}{a}(\xi,1)\underset{\sim}{a}^T(\xi,0)d\xi$$

It should be mentioned that I_{13} is the only term in the present problem which contributed to the non-self-adjointness of the problem. This is also manifested in the element matrix $\underset{\sim}{B}_7$ which is the only nonsymmetric matrix.

5.5 Element Force Vectors. These force vectors are defined in conjunction with δJ_1, δJ_2 and δJ_3 in Eqs. (5-12). Let

$$\delta J_1 = \sum_{m=1}^{K} \sum_{n=1}^{L} \left(- \frac{Tg\cos\alpha}{KL}\right) \delta \underset{\sim}{Y}^{*T}_{(m,n)} \underset{\sim}{F}_1$$

$$\delta J_2 = \sum_{m=1}^{K} \sum_{n=1}^{L} \left(- \frac{Tgm\cos\alpha}{L}\right) \delta \underset{\sim}{Y}^{*T}_{(m,n)} \underset{\sim}{F}_2$$

$$\delta J_3 = \sum_{m=1}^{K} \left(\frac{k_7}{K}\right) \delta \underset{\sim}{Y}^{*T}_{(m,L)} \underset{\sim}{F}_3$$

Then

$$\underset{\sim}{F}_1 = \int_0^1 \int_0^1 \underset{\sim}{a}(\xi,\eta)d\xi d\eta$$

$$\underset{\sim}{F}_2 = \int_0^1 \int_0^1 a(\xi,\eta)\bar{\delta}_{(m,n)}(\bar{\xi}-\xi)d\xi d\eta$$

$$F_3 = \int_0^1 Y_{(m)}(\xi)\underset{\sim}{a}(\xi,1)d\xi$$

5.6 Flow Chart of the Computer Program. As shown in Figure 5 the computations in the present program consist of two parts: data preparation and solution program.

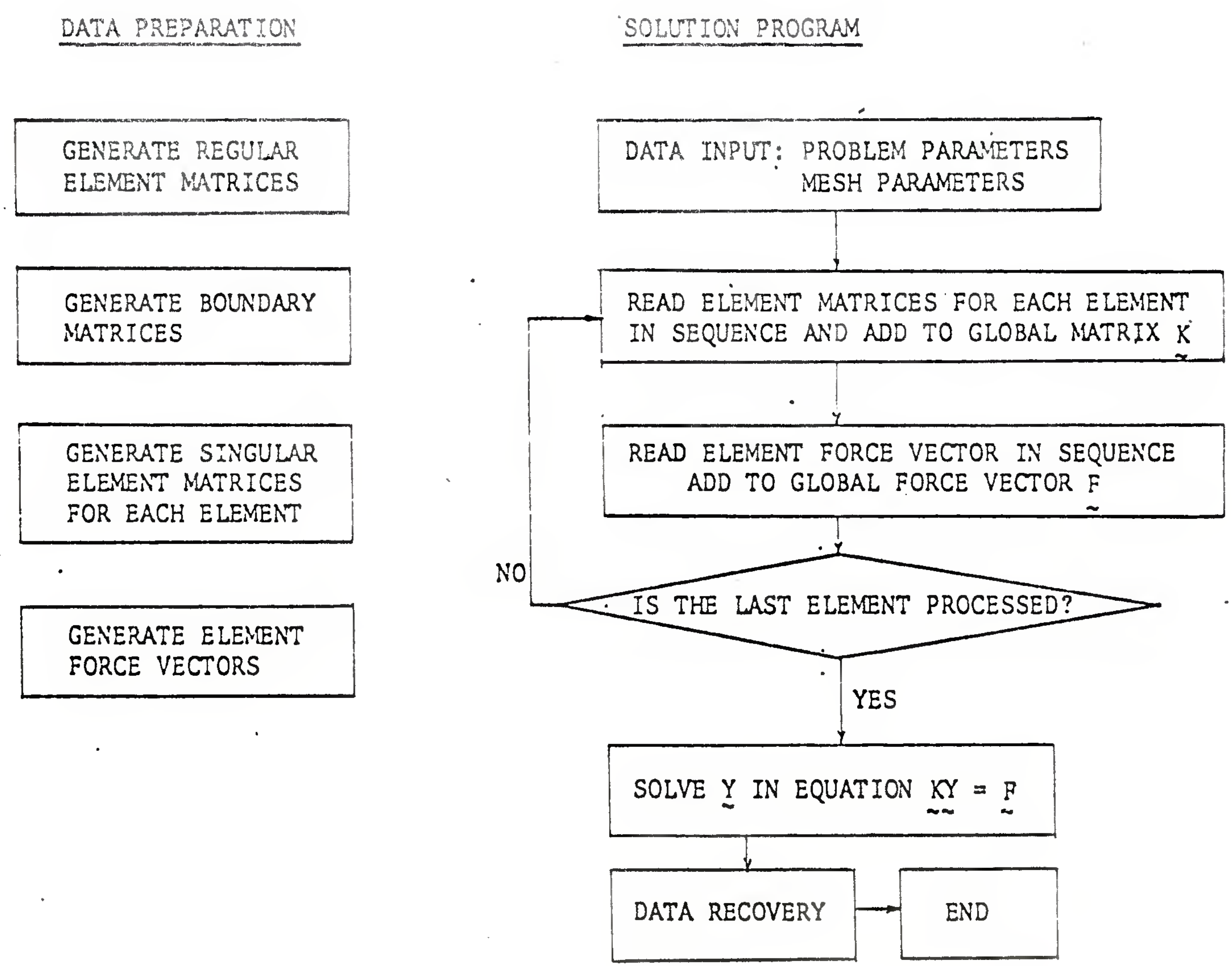


FIGURE 5. Flow Chart of the Computer Program.

6. <u>DEMONSTRATIVE RESULTS OF COMPUTATIONS</u>. Some preliminary results have been obtained by the computer program described in previous sections. Our present objective is simply to demonstrate that the concept of the unconstrained variational formulation works for the initial boundary value problem of gun dynamics, that the initial data can indeed be recovered by the unconstrained process and that the nonconservative and highly discontinuous nature of the loads can be tested in a routine and rather elegant manner. Thus the numerical values obtained are not considered final. Their verifications will be carefully performed and presented in a report to follow.

The numerical values for those nondimensional parameters listed in Section 2 are given here.

$$T = 0.07695$$

$$\bar{P} = 4.8025$$

$$m = 0.012$$

$$\beta = 308.79$$

$$\alpha = 0.5236$$

$$g = 0.020$$

These data are calculated from an approximate model for a 105 mm M68 cannon with the following material and dimensional parameters:

E (Young's modulus) $\doteq 30 \times 10^6$ psi

ρ (density) = 0.2818 lb/in^3

I.D. (of gun tube) = 105 mm = 4.13"

O.D. = 7.28" (average of 5.69" muzzle O.D. and 8.88", breech end O.D.)

$A = 28.23\ in^2$

$I = 123.60\ in^4$

$\ell = 17.5' = 210''$

$$c = \left(\frac{\rho A \ell^4}{EI}\right)^{1/2} = 0.10396 \text{ sec.}$$

m_p (projectile mass) = 210 lb

P (bore pressure) = 30,000 psi

β_p (projectile acceleration) = 500,000 ft/sec^2

The initial position of the tube is assumed to be the static deflection of the tube under gravitational force $\bar{q} = \rho A g$. Hence

$$Y(x) = \frac{q}{24}(x^4 - 4x^3 + 6x^2)$$

where $q = \frac{c^2 g}{\ell}$ and g is the gravitational acceleration. Hence q = 0.01987.

Using the data given above, the solution of a test problem is shown in Tables III and IV. The exact values of the initial data, which corresponds to the static deflection of the tube, are given in the parentheses. From these tables, it is clearly shown that the given initial data have indeed been recovered in the solutions.

TABLE III. CALCULATED $y(x,t)$ ($\times 10^{-3}$) FROM THE PRESENT COMPUTER PROGRAM (EXACT INITIAL DATA IN PARENTHESES)

t \ x	0	0.2	0.4	0.6	0.8	0.10
0	0.00000 (0.00000)	0.17477 (0.17467)	0.60803 (0.60800)	1.18801 (1.18800)	1.83469 (1.83467)	2.49996 (2.50000)
0.5T	0.00000	0.19504	0.59019	1.17861	1.82291	2.48327
T	0.00000	0.21705	0.58048	1.15876	1.77023	2.45612

TABLE IV. CALCULATED $\frac{\partial y(x,t)}{\partial x}$ ($\times 10^{-3}$) FROM THE PRESENT COMPUTER PROGRAM (EXACT INITIAL DATA IN PARENTHESES)

t \ x	0	0.2	0.4	0.6	0.8	0.10
0	0.00000 (0.00000)	1.62731 (1.62667)	2.61662 (2.61333)	3.12140 (3.12000)	3.30629 (3.30667)	3.32996 (3.33333)
0.5T	0.00000	1.63528	2.62714	3.14336	3.27633	3.31506
T	0.00000	2.06887	1.68236	3.03449	3.23987	3.55142

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